CSM—54/24

PART—II/PAPER—VI

PHYSICS

PAPER—I

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Time : 3 Hours

Full Marks : 250

The question paper contains **18 (Eighteen)** *questions in* **GROUP—A, (12)** *and* **GROUP—B, (06)** *together.*

GROUP—A

Candidates to attempt **10 *(ten)*** *questions.*

Each question carries **15** *marks.*

Marks of the bit question (a) and (b) are indicated against each.

1. *(a)* A meter stick is held at 45° with the direction of motion in a system moving with a velocity 0·8 *c*. What are the length and orientation of the stick observed from a rest frame? 8

1. *(b)* Two lumps of clay, each of rest mass *m*, collide head-on at the speed $\frac{4}{5}c$ and stick together. Find the mass of the composite lump. 7

2. *(a)* Two glasses have dispersive powers in the ratio of 2 : 3. These glasses are to be used in the manufacture of an achromatic object of focal length 20 cm. Find the focal lengths of the glasses. 8

2. *(b)* Deduce the missing orders for a double-slit Fraunhofer diffraction pattern if the slit widths are 0·16 mm and they are 0·8 mm apart. 7

3. *(a)* Write down the solution of a damped harmonic oscillator. Graphically represent various cases depending on the damping parameter. 8

3. *(b)* A particle is subjected to two simple harmonic motions along x- and y-directions with equal frequency but different amplitudes. Find the trajectory of the particle when the phase differences between two simple harmonic motions are *(i)* $\delta = 0$ and *(ii)* $\delta = \pi/2$. 7

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4. *(a)* Discuss the state of polarization when x- and y-components of electric field are given by the following equations :

(i) $E_x = E_0 \cos(\omega t + kz)$, $E_y = E_0 \cos(\omega t + kz + \pi)$

(ii) $E_x = E_0 \sin(\omega t + kz)$, $E_y = E_0 \cos(\omega t + kz)$ 8

4. *(b)* The rotation in the plane of polarization in a certain substance is 10°/cm. Find the difference between the refractive indices for right-circularly and left-circularly polarized light in the substance. (Given λ = 5893 Å) 7

5. *(a)* Determine the positions of the focal points, principal point and nodal point in the case of sphere of radius 10 cm and made of a material of μ = 1·5. Indicate the positions in a schematic diagram. 8

5. *(b)* A thin lens of focal length +16 cm is immersed in water (μ = 1·3). What is its new focal length? 7

6. *(a)* What are the major differences between interference and diffraction? 8

6. *(b)* Define coherence length. Calculate it for CO_2 laser whose line width is 10^{-5} nm at emission wave length of 10·6 micro-meter. 7

7. *(a)* Define the moment of inertia tensor. The moment of inertia tensor for a system is given by

$$M_{ij} = I_0 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Find the principal moment of inertia for that system. 8

7. *(b)* Write down the Euler equations for a force-free symmetric top. Find the solution for this top. 7

8. *(a)* Define holonomic and non-holonomic constraints with examples. The constraint equation for a system is given by

$$\dot{x}yz + x\dot{y}z + xy\dot{z} = 3$$

Find the type of constraint. 8

8. *(b)* Define degrees of freedom. Find the degrees of freedom for the following cases : 7

(i) A rigid body moving freely in 3-D space.

(ii) A solid sphere rolling without slipping on an inclined plane.

9. *(a)* The Hamiltonian of a free particle (relativistic) is given by $H = \sqrt{\vec{p}^2c^2 + m_0^2c^4}$. Construct the Lagrangian for the particle. 8

9. *(b)* Define cyclic coordinates. Explain how these are related to symmetries and conservation laws with examples. 7

10. *(a)* Calculate the magnitude and direction of Coriolis force on a 1 kg particle moving vertically upward with a velocity of 2 km/s at 60° S latitude. 8

10. *(b)* A rocket starts vertically upward with a speed v_0. Find its speed at a height h. 7

11. *(a)* Consider two inertial frames S and S'. S' is moving along $x - x'$ direction with a constant speed v. Find how different components of four-momentum vector are related in these two frames. 8

11. *(b)* Show that $\frac{d^3\vec{p}}{E}$ is Lorentz invariant. 7

12. *(a)* Construct the Lagrangian of a simple pendulum oscillating in the vertical $x - y$ plane. 8

12. *(b)* Find the Euler-Lagrange equation of motion from the Lagrangian as mentioned in previous question **12** *(a)*. Find the solution of the equation assuming angle of oscillation is small. 7

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GROUP—B

Candidates to attempt **05** ***(five)*** *questions.*

Each question carries **20** *marks.*

Marks of the bit question (a) and (b) are indicated against each.

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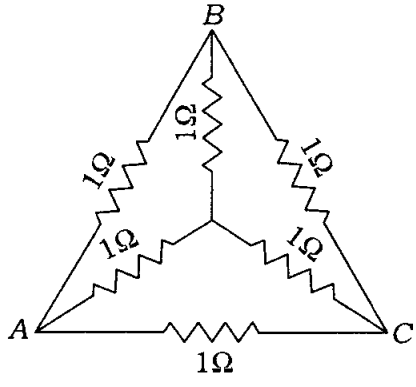
13. *(a)* N moles of an ideal gas undergo an adiabatic change from (P_i, V_i, T_i) to (P_f, V_f, T_f). Show that work done for the process is $C_v(T_i - T_f)$. 10

13. *(b)* Find the change in entropy when one mole of van der Waals gas changes from (P_1, V_1, T_1) to (P_2, V_2, T_2). 10

14. *(a)* Find the partition function for a particle at temperature T, whose dynamics is governed by the Hamiltonian $H = \begin{pmatrix} \alpha & -\beta \\ -\beta & -\alpha \end{pmatrix}$, where α and β are real numbers. 10

14. *(b)* Two Carnot engines A and B are operated in series. A receives heat at 900 K and rejects at T K. B receives heat rejected by A and in turn rejects at 400 K. Find the value of T when *(i)* work outputs of A and B are same and *(ii)* efficiencies of A and B are same. 10

15. *(a)* Find the equivalent resistance between points A and C in the following diagram : 10



15. *(b)* Draw the circuit diagrams of the series and parallel LCR circuit and construct the differential equations for the current in those circuits. 10

16. *(a)* What is the meaning of gauge fixing? Express Maxwell's equations in terms of potentials in Lorenz gauge. 10

16. *(b)* Find three different vector potentials which will produce the uniform magnetic field $\vec{B} = 2\hat{i} + 3\hat{j}$. 10

17. *(a)* What is black body? Write down Planck's law of black-body radiation and demonstrate it graphically for two different temperatures. Discuss limiting cases for high and low values of the wave length. 10

17. *(b)* A Carnot engine whose low temperature reservoir is at 7 °C has an efficiency of 40%. By how many degree should the temperature of the source to be increased to have an efficiency of 60%? 10

18. *(a)* Write Laplace's equation in spherical polar coordinates for an arbitrary potential $V(r,\theta,\varphi)$. Show that the equation can be separated into coordinates r, θ, φ if $V(r,\theta,\varphi)$ is of the form $\frac{\Phi(r)}{r} P(\theta) Q(\varphi)$, where $\Phi(r)$, $P(\theta)$ and $Q(\varphi)$ are arbitrary functions of the respective variables. 10

18. *(b)* In spherical coordinates, $V = 0$ for $r = 10$ cm and $V = 100$V for $r = 200$ cm. Assuming free space between these concentric spherical shells, find Electric field, E. 10

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SPACE FOR ROUGH WORK

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